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Cavity Pulling in Galileo Passive Hydrogen Maser

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Abstract–The paper derives the theoretical expression of the cavity pulling factor in Galileo Space Passive Hydrogen Maser, based on the 'Single frequency modulation'. The sensitivity of the cavity pulling vs. the phase selection is deduced in order to account for the experimental errors in the phase adjustments. The theoretical calculation shows the new phase setting exists to achieve a zero cavity pulling, and the preliminary experimental results comparing with the two phase selection are presented.

I. INTRODUCTION

The cavity pulling factor in a Passive Hydrogen Maser (PHM) depends on the method used to determine the atomic resonance frequency and in particular on the modulation scheme implemented [1].

The Space PHM (S-PHM) developed for Galileo is based on the 'Single frequency modulation, phase discrimination' approach [2,3]. This approach requires a proper phase selection of the synchronous detectors present in the Ultrastable Oscillator (USO) loop and in the cavity loop, in order to separate the two error signals.

The theoretical calculation of the cavity pulling factor corresponding to this technique is not available yet and is performed in this paper.

II. CALCULATION OF THE CAVITY PULLING FACTOR FOR THE CONVENTIONAL PHASE SELECTION

Fig. 1 shows a schematic diagram useful for the loop calculation. A single modulation frequency is used and the two error signals for the cavity loop and for the USO loop are discriminated by proper phase adjustment of the respective synchronous detector. For the present work we are interested essentially in the USO loop. The interrogation signal after passing through the Hydrogen (H) line discriminator, the receiver, the envelope detector, the selective amplifier and the phase shifter is synchronously detected, giving origin to the error signal of the USO loop.

In the present calculation we make use of a theoretical model for small detuning which corresponds to the situation of an operational PHM. The equations of the error signal are derived by considering the interrogation frequency detuning with respect to the H and the cavity resonance frequency, without explicitly introducing the detuning of the cavity resonance from the H line resonance. The USO loop error signal is:

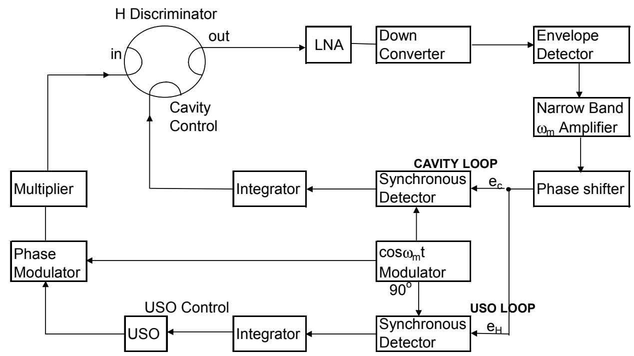

Fig. 1. Schematic diagram useful for the loop calculation

$$e_H = S_{Hx}x + S_{Hy}y \tag{1}$$

Where, e_H : Overall error signal in USO loop.

x: Normalized interrogation signal detuning from the H resonance frequency:

$$x = \frac{\omega - \omega_0}{\gamma_2} \tag{2}$$

y : Normalized cavity resonance detuning from the interrogation signal frequency:

$$y = \frac{\omega_c - \omega}{\beta} \tag{3}$$

ω : Interrogation angular frequency

ω_0 : Atomic H resonance angular frequency

ω_c : Cavity resonance angular frequency

γ_2 : Unsaturated transverse or coherence relaxation rate, i.e. half natural linewidth in rad/sec

β : Half cavity linewidth in rad/sec

S_{Hx} : The true H error signal factor in the USO loop

S_{Hy} : The spurious cavity error signal factor which is normally present in the USO loop

The expression of S_{Hx} is:

$$S_{Hx} = \frac{-J_0 J_+ r_0^2 (1+S_0)\alpha}{(1+S_0-\alpha)^2} \frac{\beta}{\sqrt{\beta^2+\omega_m^2}} \sin\left(\varphi - \arctan\frac{\beta}{\omega_m}\right) \tag{4}$$

The expression of S_{Hy} is:

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$$S_{Hy} = \frac{-J_0 J_+ r_0^2 (1+S_0)}{1+S_0-\alpha}\left[-\frac{\beta\omega_m}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{2\beta^2}{\beta^2+\omega_m^2}\right)\sin\varphi + \frac{\beta^2}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{\beta^2-\omega_m^2}{\beta^2+\omega_m^2}\right)\cos\varphi\right] \quad (5)$$

Where, J_0 : Bessel function of order 0

J_+ : Bessel function of order 1

r_0 : Amplitude transmitted by the cavity at resonance

α : Oscillation parameter

S_0 : Saturation factor at resonance

ω_m : Modulation angular frequency

φ : Phase shift in USO loop

According to [2, 3] the phase is first adjusted for obtaining a maximum H error signal in the USO loop, for nearly tuned cavity, in a second step the reference phase of the cavity loop synchronous detector is shifted of 90 degrees resulting in a pure cavity error signal in the cavity loop.

From (4), the true H error signal in the USO loop is maximized with the phase shift:

$$\varphi_a = \arctan\left(\frac{\beta}{\omega_m}\right) - \frac{\pi}{2} \quad (6)$$

Typically for S-PHM with $\beta = 6.28\times10^5$ (for $Q_c = 7000$) and $\omega_m = 7.85\times10^4$ rad/sec, ϕ_a = - 0.124 rad (i.e. - 7.1°).

Replacing this value in (4-5), we have

$$S_{Hx} = \frac{J_0 J_+ r_0^2 (1+S_0)\alpha}{(1+S_0-\alpha)^2}\frac{\beta}{\sqrt{\beta^2+\omega_m^2}} \quad (7)$$

$$S_{Hy} = \frac{-J_0 J_+ r_0^2 (1+S_0)}{1+S_0-\alpha}\left[\frac{\beta\omega_m}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{2\beta^2}{\beta^2+\omega_m^2}\right)\cos\left(\arctan\frac{\beta}{\omega_m}\right) + \frac{\beta^2}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{\beta^2-\omega_m^2}{\beta^2+\omega_m^2}\right)\sin\left(\arctan\frac{\beta}{\omega_m}\right)\right] \quad (8)$$

The condition that the USO loop is closed provides a null of the error signal:

$$S_{Hx}x + S_{Hy}y = 0 \quad (9)$$

The origin of the cavity pulling effect is the presence of the spurious cavity error signal in (9).

The interrogation frequency i.e. the maser frequency which is satisfying (9) has the following offset $\Delta\omega_0$ from the atomic resonance frequency:

$$\Delta\omega_0 = \frac{\alpha\beta^2+\omega_m^2(1+S_0)}{\alpha(\beta^2+\omega_m^2)}\frac{\gamma_2}{\beta}\Delta\omega_c = \frac{\alpha\beta^2+\omega_m^2(1+S_0)}{\alpha(\beta^2+\omega_m^2)}\frac{Q_c}{Q_0}\Delta\omega_c \quad (10)$$

Where, Q_c : Quality factor of the cavity, $Q_c = \dfrac{\omega_c}{2\beta}$,

Q_0 : Quality factor of the atomic unsaturated line, $Q_0 = \dfrac{\omega_0}{2\gamma_2}$,

and $\Delta\omega_c$ is the offset of the cavity resonance frequency from the atomic resonance.

For small cavity detuning, the maser frequency offset is proportional to the cavity resonance frequency offset.

The cavity pulling factor F is:

$$F = \frac{\alpha\beta^2+\omega_m^2(1+S_0)}{\alpha(\beta^2+\omega_m^2)}\frac{Q_c}{Q_o} = C\frac{Q_c}{Q_o} \quad (11)$$

For the usual operating conditions of PHM (α=0.4 and $\beta >> \omega_m$), the cavity pulling factor is approximately equal to $\dfrac{Q_c}{Q_0}$, i.e. similar to the expression of an active maser. Fig. 2 gives the factor C v/s α and S_0 for above typical values of β and ω_m.

Replacing the typical values of the Galileo S-PHM parameters: $\alpha = 0.38$, $S_0 = 0.4$, $Q_0 = 5\times10^8$ i.e. $\gamma_2 = 8.8$ rad/sec, (11) gives C=1.04 ($F = 1.46\times10^{-5}$), which is in good agreement with our experimentally measured value of 1.45×10^{-5}.

III. Sensitivity of the Cavity Pulling to the Phase Error

In the following the theoretical expression of the sensitivity of the cavity pulling v/s the phase setting is derived in order to evaluate the error in the cavity pulling measurements due to errors in the phase adjustments.

In case the phase shift in H loop is $\phi_a + \Delta\phi$ ($\Delta\phi$: Phase error), (7) and (8) become

$$S_{Hx} = \frac{J_0 J_+ r_0^2 (1+S_0)\alpha}{(1+S_0-\alpha)^2}\frac{\beta}{\sqrt{\beta^2+\omega_m^2}}\cos(\Delta\phi) \quad (12)$$

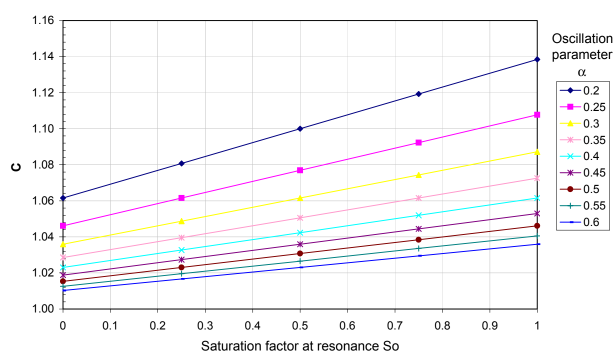

Fig.2. Factor C v/s S_0 for several values of α (with $\beta = 6.28\times10^5$ and $\omega_m = 7.85\times10^4$ rad/sec)

$$S_{Hy} = \frac{-J_0 J_+ r_0^2 (1+S_0)}{1+S_0-\alpha}\left[\frac{\beta\omega_m}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{2\beta^2}{\beta^2+\omega_m^2}\right)\cos\left(\arctan\frac{\beta}{\omega_m}+\Delta\phi\right)\right.$$
$$\left.+\frac{\beta^2}{\beta^2+\omega_m^2}\left(\frac{1+S_0}{1+S_0-\alpha}-\frac{\beta^2-\omega_m^2}{\beta^2+\omega_m^2}\right)\sin\left(\arctan\frac{\beta}{\omega_m}+\Delta\phi\right)\right] \tag{13}$$

Solving (9) gives the expression of the cavity pulling factor F' v/s $\Delta\phi$:

$$F' = \frac{\gamma_2 \sec(\Lambda\phi)}{\alpha\beta\left(\beta^2+\omega_m^2\right)^{3/2}}\left\{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2\right]\cos\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]\right.$$
$$+\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)\right]\sin\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]$$
$$\left.+\omega_m S_0\left[\left(\omega_m^2-\beta^2\right)\cos\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]+2\beta\omega_m\sin\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]\right]\right\} \tag{14}$$

According to (11) and (14), The cavity pulling factor relative error is

$$\frac{F'}{F} = \frac{\sec(\Delta\phi)}{\sqrt{\beta^2+\omega_m^2}\left[\alpha\beta^2+\omega_m^2(1+S_0)\right]}\left\{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2\right]\cos\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]\right.$$
$$+\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)\right]\sin\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]$$
$$\left.+\omega_m S_0\left[\left(\omega_m^2-\beta^2\right)\cos\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]+2\beta\omega_m\sin\left[\Delta\phi+\arctan\left(\frac{\beta}{\omega_m}\right)\right]\right]\right\} \tag{15}$$

For the typical values of β, ω_m, α and S_0,

$$\frac{F'}{F} = 1+0.317\tan(\Delta\phi) \tag{16}$$

Note the unit of $\Delta\phi$ is 'rad' in above equation.

The sensitivity of the relative error $\frac{F'}{F}$ to the phase error $\Delta\phi$ [degree] is obtained by the derivative of (16):

$$S = \frac{5.54\times10^{-3}}{\cos^2\left(\frac{\pi\cdot\Delta\phi}{180}\right)} \tag{17}$$

The relative error v/s the phase offset $\Delta\phi$ in range of $\pm15^{o}$ is shown in Fig. 3. Over this range the sensitivity S of the cavity pulling relative error is approximately constant and equal to 5.59×10^{-3}/degree. At $\Delta\phi$=0, i.e. phase selection of ϕ_a, the cavity pulling factor is $F' = F$.

IV. Possibility of Zero Cavity Pulling for a New Phase Selection

With the phase selection of ϕ_a given by (6), the maximum H error signal obtained in the USO loop is actually contaminated by the presence of a cavity error signal. The theoretical calculation shows that a specific phase shift exists, for which the USO loop error signal can be truly independent

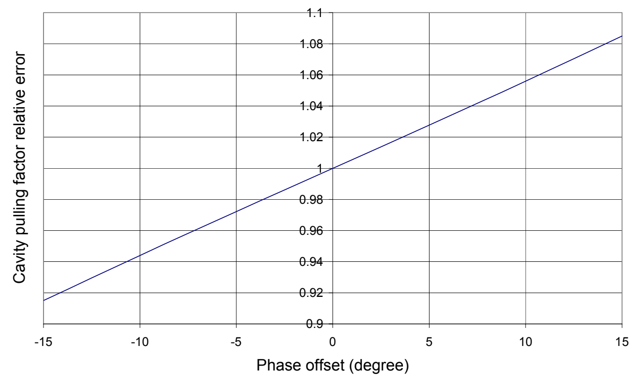

Fig. 3. Cavity pulling factor relative error $\frac{F'}{F}$ v/s the phase offset $\Delta\phi$ [degree] (assuming $\beta = 6.28\times10^5$, $\omega_m = 7.85\times10^4$ rad/sec, $\alpha = 0.38$, $S_0 = 0.4$, ϕ_a = - 7.1°)

from the cavity frequency, i.e. the USO loop error signal is a pure H signal.

Equation (5) can be also written in the following form:

$$S_{Hy} = \frac{J_0 J_+ r_0^2 (1+S_0)}{1+S_0-\alpha}\frac{\beta}{\beta^2+\omega_m^2}\sqrt{\frac{\alpha^2\beta^2+\omega_m^2+2\omega_m^2 S_0+\omega_m^2 S_0^2}{(1+S_0-\alpha)^2}}$$
$$\cdot\sin\left[\varphi-\arctan\frac{\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)+2\omega_m^2 S_0\right]}{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2-\left(\beta^2-\omega_m^2\right)S_0\right]}\right] \tag{18}$$

To obtain the spurious cavity error signal factor $S_{Hy}=0$, the phase shift φ_b must be:

$$\varphi_b = \arctan\frac{\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)+2\omega_m^2 S_0\right]}{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2-\left(\beta^2-\omega_m^2\right)S_0\right]} \tag{19}$$

Replacing this value in (4) gives:

$$S_{Hx} = \frac{-J_0 J_+ r_0^2 (1+S_0)\alpha}{(1+S_0-\alpha)^2}\frac{\beta}{\sqrt{\beta^2+\omega_m^2}}$$
$$\cdot\sin\left(\arctan\frac{\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)+2\omega_m^2 S_0\right]}{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2-\left(\beta^2-\omega_m^2\right)S_0\right]}-\arctan\frac{\beta}{\omega_m}\right) \tag{20}$$

This is the pure H error signal of the USO loop, i.e.:

$$e_H = S_{Hx}x \tag{21}$$

and the cavity pulling effect is zero.

Comparing (20) with (7), the H error signal is reduced from the maximum signal by the ratio

$$R = -\sin\left(\arctan\frac{\beta\left[2\omega_m^2+\alpha\left(\beta^2-\omega_m^2\right)+2\omega_m^2 S_0\right]}{\omega_m\left[(2\alpha-1)\beta^2+\omega_m^2-\left(\beta^2-\omega_m^2\right)S_0\right]}-\arctan\frac{\beta}{\omega_m}\right) \tag{22}$$

Replacing the typical values of the parameters, φ_b=-1.388 rad (i.e.-79.5°) and R=0.3.

The price to be paid for obtaining a pure H error signal in the USO loop is the reduction of the H error signal by a factor of 3 compared to its maximum value.

V. PRACTICAL DETERMINATION OF THE NEW PHASE SETTING

The experimental procedure for deriving φ_b is as follows:

Both USO and cavity loops are open, detuning in both loops is assumed small. This can be done by a preliminary close loop operation using the standard configuration or by tuning the interrogation frequency and cavity frequency using an accurate reference frequency source.

1) Detune the interrogation frequency in order to obtain a definite H error signal.

2) Modulate the cavity frequency via the cavity varactor.

3) Change smoothly the phase shift of the USO loop in order to minimize the modulation on the H error signal produced by modulating the cavity frequency. Theoretically a phase shift exits for which the modulation of the cavity frequency does not produce a modulation of the H error signal. This is the value of φ_b for the USO loop.

A preliminary experiment has been made in order to evaluate the cavity pulling effect with the different phase selections. The results show that in the condition of the experiment with the conventional phase selection the measured cavity pulling factor is 1.05×10^{-5}, and by using the new phase selection the cavity pulling factor is reduced to 2.41×10^{-6}. Further work is in progress towards the realisation of the zero cavity pulling.

VI. CONCLUSION

The theoretical expression of the cavity pulling factor as a function of the S-PHM parameters for the normal phase selection giving the maximum H error signal is derived, which is in good agreement with the measurement. The cavity pulling measurement is affected by error in the phase adjustment and the sensitivity of cavity pulling factor is about 5.6×10^{-3}/degree for the phase error within $\pm 15^{\circ}$.

A null cavity pulling for a new selection of the phase setting is deduced theoretically. The preliminary experimental result confirms the feasibility of this new approach.

This new phase setting can improve significantly the long term performance of a PHM even if the H error signal is reduced by a factor of 3.

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